

# Cosmological aspects of a vector field model

S. Davood Sadatian<sup>1</sup>

*Department of Physics, Faculty of Basic Sciences,  
University of Neyshabur,  
P. O. Box 91136-899, Neyshabur, Iran*

## Abstract

We have studied a DGP-inspired braneworld scenario where the idea of Lorentz invariance violation has been combined into a specifying preferred frame that embed a dynamical normal vector field to brane. We propose the Lorentz violating DGP brane models with enough parameters can explain crossing of phantom divide line. Also we have considered the model for proper cosmological evolution that is according to the observed behavior of the equation of state. In other view point, we have described a Rip singularity solution of model that occur in this model.

**PACS:** 04.50.+h, 98.80.-k

**Key Words:** DGP Brane Cosmology, Braneworld Cosmology, Scaler-Vector-Tensor Theories, Lorentz Invariance Violation.

---

<sup>1</sup>sd-sadatian@um.ac.ir

# 1 Introduction

According observation data from WMAP, universe accelerated expanding at the current epoch [1,2,3,4,5]. These astrophysical observations also show that the universe is spatially, and is composed of about 27% of dark matter, and 73% of homogeneously distributed new type of negative pressure matter, known as dark energy, that lead to the current accelerated expansion in universe. From another view point, theories of extra dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have attracted some attentions. Particularly, the model proposed by Dvali, Gabadadze and Porrati (DGP) [6,7,8] is different from others. In this regard, DGP model also predicts deviations from the standard 4-dimensional gravity in over large distances. However, impact of Lorentz invariance violating models (LIV) on cosmology has been studied in [9,10,11]. This models has been used in the context of scalar-vector-tensor theories [9,11]. One important observation has been made in reference [11,12] which considered accelerated expansion and crossing of phantom divide line with one minimally coupled scalar field in the attendance of a Lorentz invariance violating vector field. As one has shown in [13], quintessence model with a single minimally coupled scalar field has not the capability to describe crossing of phantom divide line,  $\omega = -1$ . But, a single non-minimally coupled scalar field is suitable to cross the phantom divide line [14].

In following, we study a new method of performing local Lorentz violation in a gravitational setup to make the existence of a tensor field with a non-vanishing expectation value, and then add this tensor to gravity or matter and scalar fields. The example of this approach is to used a single time-like vector field with unit norm. This vector field choose a preferred frame at each point in space-time and a matter field coupled to it will endure a violation of local Lorentz invariance. A method of this model was introduced Kostelecky and Samuel in [15]. Here we choose the vector to have dynamics, and fixed norm in the action.

Our procedure in this paper is deriving the basic equations of motion for the most general theory of a time-like vector field  $u^\mu$  with an action  $S = S_{Bulk} + S_{Brane}$  where  $S_{Brane} = S_{EH} + S_\phi + S_m + S_u$  [11]. Then with numerical calculations, we study some cosmological aspects in this model in present the vector field in our formalism. In this regard, we study other solutions admitted as Rip singularity, that occur in the condition  $\omega < -1$  increases rapidly. However, it possible different types of singularity, depending energy density and scale factor how increases with time[16-19,31-36]. In following we just focus on Big or Little singularity and other cases of singularity maybe appear in this model with tuning parameters space.

## 2 A Lorentz Violating DGP Brane Model

As we shown in [11], for study Lorentz violation, we assume a vector  $u^\mu$  in the extra dimension. So, this local frame at space-time is unavoidably selected as the preferred frame. However the action of the Lorentz violating DGP scenario in the presence of a minimally coupled scalar field and vector field on the Brane can be written as the sum of two separate parts

$$S = S_{Bulk} + S_{Brane}, \quad (1)$$

where  $S_{Bulk}$  and  $S_{Brane} = S_{EH} + S_\phi + S_m + S_u$  are

$$S_{Bulk} = \int d^5x \frac{m_4^3}{2} \sqrt{-g} \mathcal{R}, \quad (2)$$

$$S_{Brane} = \left[ \int d^4x \sqrt{-q} \left( \frac{m_3^2}{2} R[q] - \frac{1}{2} q^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) + m_4^3 \overline{K} + \mathcal{L}_m + \right. \right. \\ \left. \left. [-\beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu - \beta_3 (\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u^\nu \nabla_\mu u^\alpha \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1)] \right) \right]_{y=0}. \quad (3)$$

$y$  is coordinate of the fifth dimension,  $m_4^3$  point to the constant in the Bulk and  $m_3^2$  used for Brane, for more details see [11]. We consider brane is located at  $y = 0$ .  $g_{AB}$  is five dimensional bulk metric with Ricci scalar  $\mathcal{R}$ , also  $q_{\mu\nu}$  is induced metric on the brane with induced Ricci scalar  $R$ .  $g_{AB}$  and  $q_{\mu\nu}$  are connected via  $q_{\mu\nu} = \delta_\mu^A \delta_\nu^B g_{AB}$ . This action is suitable for add any non-gravitational degrees of freedom in the model of Lorentz violating scalar-tensor-vector theory. In other hand, we take  $u^\mu u_\mu = -1$  and the expectation value of vector field  $u^\mu$  is  $\langle 0 | u^\mu u_\mu | 0 \rangle = -1$  [20].  $\beta_i(\phi)$  ( $i = 1, 2, 3, 4$ ) are temporary parameters with dimension of mass squared,  $\lambda$  is also a Lagrange multiplier field. Note that  $\sqrt{\beta_i}$  are mass scale of violating Lorentz invariance [9,20,21].

A motivation for choosing the vector action in term (3) discussed in Ref. [20]. Also cosmological consequences of this action are studied in Ref. [9,11,21].

In the action (3) ordinary matter part is shown by Lagrangian  $\mathcal{L}_m \equiv \mathcal{L}_m(q_{\mu\nu}, \psi)$  where  $\psi$  is matter field and its energy-momentum tensor is [22]

$$T_{\mu\nu} = -2 \frac{\delta \mathcal{L}_m}{\delta q^{\mu\nu}} + q_{\mu\nu} \mathcal{L}_m. \quad (4)$$

The scalar field Lagrangian,  $\mathcal{L}_\phi = -\frac{1}{2} q^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi)$ , gives the following energy-momentum tensor

$$\tau_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} q_{\mu\nu} (\nabla \phi)^2 - q_{\mu\nu} V(\phi). \quad (5)$$

The energy-momentum tensor of vector field also is by usual formulate

$$T_{\mu\nu}^{(u)} = -2 \frac{\delta \mathcal{L}^{(u)}}{\delta q^{\mu\nu}} + q_{\mu\nu} \mathcal{L}^{(u)}. \quad (6)$$

The Bulk-brane Einstein's equations obtained from action (1) are given by

$$m_4^3 \left( \mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} \right) + m_3^2 \delta_A^\mu \delta_B^\nu \left( R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R \right) \delta(y) = \delta_A^\mu \delta_B^\nu \Upsilon_{\mu\nu} \delta(y) \quad (7)$$

where  $\square^{(4)}$  is 4-dimensional(brane) d'Alembertian and  $\Upsilon_{\mu\nu} = T_{\mu\nu} + \tau_{\mu\nu} + T_{\mu\nu}^{(u)}$ .

From equation (7) we find

$$G_{AB} = \mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} = 0 \quad (8)$$

and

$$G_{\mu\nu} = \left( R_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R \right) = \frac{\Upsilon_{\mu\nu}}{m_3^2} \quad (9)$$

for bulk and brane respectively. We use the following line element to derive cosmological equations of our model

$$ds^2 = q_{\mu\nu} dx^\mu dx^\nu + b^2(y, t) dy^2 = -n^2(y, t) dt^2 + a^2(y, t) \gamma_{ij} dx^i dx^j + b^2(y, t) dy^2 \quad (10)$$

we use  $n(0, t) = 1$  and  $\mathcal{N} = 1$  after the variation [11].

Now we can obtain components of the total energy-momentum tensor as follow

$$\rho = \rho_m + \rho_u + \rho_\phi \quad (11)$$

and

$$p = p_m + p_u + p_\phi \quad (12)$$

Here we take the energy-momentum tensor for the matter as a perfect fluid with energy density  $\rho_m$  and pressure  $p_m$  is  $T_{\mu\nu} = (\rho_m + p_m) N_\mu N_\nu + p_m q_{\mu\nu}$ , where  $N_\mu$  is a unit time-like vector field illustrate the fluid four-velocity. We also assume a usual equation of state for the fluid as  $p_m = (\gamma - 1) \rho_m$  that  $1 \leq \gamma \leq 2$ . Energy density and pressure of minimally coupled scalar field are taken as

$$\rho_\phi = \left[ \frac{1}{2} \dot{\phi}^2 + n^2 V(\phi) \right]_{y=0}, \quad (13)$$

and

$$p_\phi = \left[ \frac{1}{2n^2} \dot{\phi}^2 - V(\phi) \right]_{y=0}, \quad (14)$$

where a dot point to the derivative respect to cosmic time  $t$ . The stress-energy for the vector field also assume the form of a perfect fluid, with an energy density determined by [10]

$$\rho_u = -3\beta H^2 \quad (15)$$

and a pressure

$$p_u = \beta H^2 \left[ 3 + 2 \frac{\dot{H}}{H^2} + 2 \frac{\dot{\beta}}{H\beta} \right] \quad (16)$$

where we have assume the parameter  $\beta \equiv (\beta_1 + 3\beta_2 + \beta_3)$  and  $H = \frac{\dot{a}(0,t)}{a(0,t)}$  is Hubble parameter. In the absence of vector field, all  $\beta_i = 0$ , In following, we stress for obtain equation of state in general case one should varying the action (1), that It has been calculated in Ref[9,10 11,12,15,20, 21 and 22]. Here we just used their results and summarized calculations. Hence, we write the general equation of state which takes the following form

$$\omega = \left[ \frac{p_m + \beta H^2 \left[ 3 + 2 \frac{\dot{H}}{H^2} + 2 \frac{\dot{\beta}}{H\beta} \right] + \frac{1}{2n^2} \dot{\phi}^2 - V(\phi)}{\rho_m - 3\beta H^2 + \frac{1}{2} \dot{\phi}^2 + n^2 V(\phi)} \right]_{y=0} \quad (17)$$

and for obtain generalized Friedmann equations, we use the effective Einstein equation (for more details see[11,23]), therefore we have

$$3 \left( H^2 + \frac{k}{a^2} \right) = \mathcal{E}_0 + \frac{1}{12m_4^6} \left[ \rho_m - 3\beta H^2 + \frac{1}{2} \dot{\phi}^2 + n^2 V(\phi) - 3m_3^2 \left( H^2 + \frac{k}{a^2} \right) \right]^2. \quad (18)$$

As we calculated in above equations, there are many parameters, so we expect the Lorentz violating brane model, suitable for study the current stage acceleration of the universe and other cosmological aspects with fine tuning parameters.

### 3 Setup for useful equations

In following, we consider scalar equation of state  $\omega_\phi(t)$  which is a candidate for the dark energy in future discussions. In this regard, the equation of state parameter on the brane given by

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \left[ \frac{\frac{1}{2n^2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + n^2 V(\phi)} \right]_{y=0} \quad (19)$$

Now we obtain scalar field of equation of state in two ways. First, we have Friedmann equation from equation (18) as

$$H^2 = \frac{\mathcal{E}_0}{3a^4} + \frac{1}{36m_4^6} \left[ -3\beta H^2 + \frac{1}{2} \dot{\phi}^2 + V(\phi) - 3m_3^2 H^2 \right]^2 \quad (20)$$

where  $k = 0$  and we assume non matter contain on the model that means  $\rho_m = 0$  and  $p_m = 0$ , where  $\dot{\mathcal{E}}^0_0 + 4H\mathcal{E}^0_0 = 0$  and  $\mathcal{E}^0_0 = \frac{\mathcal{E}_0}{a^4}$  with  $\mathcal{E}_0$  as an integration constant[23]. Then we obtain dynamic of scalar field from above equation

$$\dot{\phi}^2 = 6H^2(\beta + m_3^2) - 2V(\phi) + 12m_4^3\epsilon\sqrt{H^2 + \frac{\mathcal{E}_0}{3a^4}} \quad (21)$$

where  $\epsilon = \pm 1$  has main role in next calculations. For determine dynamic of scalar field, we use to a well-known potential: models with potential of the form  $V(\phi) = \lambda'\phi^2$  [24].

Second, Such as we see equation (21) depend on potential functions, therefore we will obtain the scalar field of equation of state in our model that independent of potentials. So the energy equation for vector field taken as

$$\dot{\rho}_u + 3H(\rho_u + p_u) = +3H^2\dot{\beta} \quad (22)$$

and for the scalar field

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -3H^2\dot{\beta}. \quad (23)$$

The total energy equation in presence of both the vector and the scalar fields is

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi) \quad (24)$$

Now we obtain dynamic of scalar field by differentiating equation (13) with respect to  $t$  and using equation (23) then we have

$$\ddot{\phi} + 3H\dot{\phi} + 3H^2\beta_{,\phi} + V_{,\phi} = 0 \quad (25)$$

and by differentiating equation (20) with respect to  $t$  and using equation (25) we have

$$\dot{\phi} = \mp 2m_4^3 \left( \frac{2H_{,\phi}}{\dot{\phi}H} + \frac{12\mathcal{E}_0}{\dot{\phi}^2 \dot{a} a^3} \right)^{\frac{1}{2}} - 2H\beta_{,\phi} - 2\beta H_{,\phi} - m_3^2 \frac{H_{,\phi}}{H} \quad (26)$$

where we assume  $H$  and  $\beta$  depended on  $\phi$ . For simplicity in following we take  $\mathcal{E}_0 = 0$ . In other hand, with substituting Equation (26) into the Friedmann equation (20) obtain the potential of the scalar field as

$$V(\phi) = \frac{1}{2} \frac{12H + 6m_3^2 H^2 m_4^3 - \dot{\phi}^2 m_4^3 + 6H^2 m_4^3 \beta}{m_4^3} \quad (27)$$

and

$$V(\phi) = \frac{1}{2} \frac{-12H + 6m_3^2 H^2 m_4^3 - \dot{\phi}^2 m_4^3 + 6H^2 m_4^3 \beta}{m_4^3} \quad (28)$$

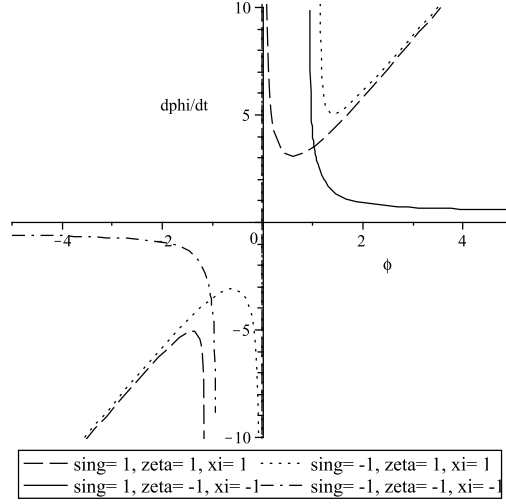


Figure 1: Variation of  $\frac{d\phi}{dt}$  relative to  $\phi$  in equation (29) for different values of  $\xi$  and  $\zeta$  according ansatz  $H = H_0\phi^\zeta$  and  $\beta(\phi) = m\phi^\xi$ .

Also factor  $\dot{\phi}$  from equation (26) give as

$$\begin{aligned} \dot{\phi} = & 1 + 2\beta_{,\phi}\beta H_{,\phi} + .6\beta_{,\phi}H_{,\phi} + .6\beta H \times \\ & \sqrt[3]{\left(10H_{,\phi} + 4 + 4\beta H_{,\phi} + 5\beta_{,\phi} + 4\beta_{,\phi}\beta + 80\beta H + A\right)} \\ & \frac{H}{\times \mathcal{O}(\phi)} \end{aligned} \quad (29)$$

where  $A = 2\sqrt{100 + 10H_{,\phi} + 30\beta + 40\beta_{,\phi} + 30\beta_{,\phi}\beta + 10\beta H \epsilon' H}$  and  $\epsilon' = \pm 1$ . We emphasize that equation (26) have several solutions, here we choose a real solution of them. Besides, equation (29) has not the analytical solution that we exactly use it for cosmological consideration. Hence, we just study some phenomenological aspects from Figure 1 and choose a best fit scalar field with equation (29) for analytical consideration.

In follows, we calculate scalar field of equation of state with using equation (20). Therefore we have two equations for  $H^2$  as

$$H^2 = \frac{1}{3} \frac{\rho_\phi m_3^2 + \beta \rho_\phi + 6 m_4^6 + 2 \sqrt{3 m_4^6 \rho_\phi m_3^2 + 3 m_4^6 \beta \rho_\phi + 9 m_4^{12}}}{2 \beta m_3^2 + \beta^2 + m_3^4} \quad (30)$$

and

$$H^2 = \frac{1}{3} \frac{\rho_\phi m_3^2 + \beta \rho_\phi + 6 m_4^6 - 2 \sqrt{3 m_4^6 \rho_\phi m_3^2 + 3 m_4^6 \beta \rho_\phi + 9 m_4^{12}}}{2 \beta m_3^2 + \beta^2 + m_3^4}. \quad (31)$$

For each branch of  $H^2$  means equation (30) and (31) with using equation (23) we obtain equation

$$E1 + E2(1 + \omega_\phi) = -\dot{\beta}E3 \quad (32)$$

where

$$E1 = \frac{\dot{\rho}_\phi}{\rho_\phi} = \frac{\dot{H}}{H} + \frac{\dot{\beta} H m_4^3 + \beta \dot{H} m_4^3 + m_3^2 \dot{H} m_4^3}{2 \epsilon_4 + \beta H m_4^3 + m_3^2 H m_4^3} \quad (33)$$

and

$$E2 = \frac{3 \epsilon_1 \left( m_4^3 + \epsilon_2 \sqrt{\frac{m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2)}{m_4^3}} \right)}{(\beta + m_3^2)} \quad (34)$$

and

$$E3 = \frac{2 \epsilon_3 \sqrt{m_4^3 \left( m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2) \right) m_4^3}}{H (m_4^3 (\beta + m_3^2) H + 2 \epsilon_4) (\beta + m_3^2)^2} + \frac{H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2) + 2 m_4^9}{H (m_4^3 (\beta + m_3^2) H + 2 \epsilon_4) (\beta + m_3^2)^2} \quad (35)$$

where  $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \pm 1$ . We can calculate  $\omega_\phi$  after simplify from equation (32) as

$$\begin{aligned} \omega_\phi = 1/6 & \left[ -6 \epsilon_2 (\beta + m_3^2) H \epsilon_1 \left( 1/2 m_4^3 (\beta + m_3^2) H + \epsilon_4 \right) \times \right. \\ & \sqrt{\frac{m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2)}{m_4^3}} - \\ & 2 \beta_{,\phi} \dot{\phi} \epsilon_3 \sqrt{m_4^3 \left( m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2) \right) m_4^3} - \\ & 3 m_4^3 (\beta + m_3^2)^2 \left( 2/3 \beta_{,\phi} \dot{\phi} + \epsilon_1 m_4^3 \right) H^2 - \\ & 6 \left( \left( 1/3 H_{,\phi} \dot{\phi} m_3^4 + 2/3 H_{,\phi} \dot{\phi} \beta m_3^2 + \epsilon_1 \epsilon_4 + 1/3 \beta^2 H_{,\phi} \dot{\phi} \right) m_4^3 + 1/3 \beta_{,\phi} \dot{\phi} \epsilon_4 \right) (\beta + m_3^2) H - \\ & 2 \left( m_4^9 \beta_{,\phi} + H_{,\phi} \epsilon_4 (\beta + m_3^2)^2 \right) \dot{\phi} \left[ \left( m_4^3 + \epsilon_2 \sqrt{\frac{m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2 H \epsilon_4 (\beta + m_3^2)}{m_4^3}} \right)^{-1} \right. \\ & \left. \left. \times (\beta + m_3^2)^{-1} H^{-1} \epsilon_1^{-1} \left( 1/2 m_4^3 (\beta + m_3^2) H + \epsilon_4 \right)^{-1} \right] \right] \quad (36) \end{aligned}$$

In following we study our models by using above equations for obtain some constraint of parameters that be according to observation data and also discuss about other cosmological aspects.



## 4 Fine tuning parameters

In this section we consider our equation that obtained in above section for determine some cosmological aspects. Therefore we need to solve equation (36) to study crossing of phantom divide barrier  $\omega_\phi$  in model. In first stage we should obtain dynamics of scalar field  $\phi$  with equation (21). This will be achieved only if the Hubble parameter  $H(\phi(t))$  and the vector field coupling,  $\beta(\phi(t))$  are known. Hence, our strategy is to choose some cases of the Hubble parameter  $H(\phi(t))$  and the vector field coupling  $\beta(\phi(t))$  for considering possible crossing of phantom divide barrier (PDL) and Rip singularity in this model. Also we should obtain suitable domains of parameter space which have the capability to explain Rip singularity and crossing of phantom divide line by equation of state parameter.

In following, we take a general case of the vector field coupling and the Hubble parameter that are functions of scalar field  $\phi$

$$H = H_0\phi^\zeta, \quad \beta(\phi) = m\phi^\xi \quad (37)$$

where  $H_0$  and  $m$  are positive constant parameters. Here we used anzats (37) because considered some cases of the solution and verified the stability in previous works [12]. Also other authors [10 and references therein] have been used anzats (37) for some cosmological solutions, for example, late time acceleration, deceleration parameters and etc. So let us using equation (21) for obtain dynamic of scalar field equation with assuming case (37) as

$$\dot{\phi} = 6H_0^2\phi^{2\zeta}(m\phi^\xi + m_3^2) - \lambda'\phi^2 + 12m_4^3\epsilon H_0\phi^\zeta \quad (38)$$

for potential  $V(\phi) = \lambda'\phi^2$ .

If we use a simple case from a acceptable range of  $\xi$  and  $\zeta$  for analytical solution of scalar field equation (38), we obtain analytical solution for dynamic of scalar filed equation as follows

$$\begin{aligned} \phi(t) = \frac{1}{2} \Bigg[ & \left( 36m_4^6\epsilon^2 H_0^2 e^{2A_0\sqrt{6H_0^2 m_3^2 - \lambda'}} + 9H_0^4 m^2 e^{2A_0\sqrt{6H_0^2 m_3^2 - \lambda'}} - \right. \\ & 12e^{(t+A_0)\sqrt{6H_0^2 m_3^2 - \lambda'}} \sqrt{6H_0^2 m_3^2 - \lambda'} m_4^3 \epsilon H_0 + 6e^{2t\sqrt{6H_0^2 m_3^2 - \lambda'}} H_0^2 m_3^2 - \\ & \left. e^{2t\sqrt{6H_0^2 m_3^2 - \lambda'}} \lambda' - 6e^{(t+A_0)\sqrt{6H_0^2 m_3^2 - \lambda'}} \sqrt{6H_0^2 m_3^2 - \lambda'} H_0^2 m + 36H_0^3 m m_4^3 \epsilon e^{2A_0\sqrt{6H_0^2 m_3^2 - \lambda'}} \right) \\ & \left. e^{-(t+A_0)\sqrt{6H_0^2 m_3^2 - \lambda'}} \right] \left[ \left( 6H_0^2 m_3^2 - \lambda' \right)^{3/2} \right]^{-1} \end{aligned} \quad (39)$$

where we take  $\xi = -1$ ,  $\zeta = 1$  and  $A_0$  is an integration constant.

In the following we consider equation of state (36) with using equation (39) for exact dynamic of equation of state, Then study crossing phantom divided line and other options in our model. The result of our numerical calculation has been shown in figures 2-5 and 6-9. The summarize of results given in table 1. According figures 2-5 and table 1, Our model can explain crossing phantom divided, that is relevant to observational data. In other hand, one has been discussed in [29-36] about what conditions lead to a Rip singularity solutions, now we consider, can this model predict a Rip singularity in context. we note that just two cases of singularity means Big and Little singularity study in following.

Therefore, as shown in figures 6-9, we provide suitable conditions for appear Rip singularity (this situation occur in  $\epsilon = +1$  branch). As see in a numerical solution, this situations appear in the phantom barrier, in this case the equation of state increase rapidly in finite or infinite time. Hence we can consider the solution in two major cases as known Little and Big singularities. In fact we can take a dynamical explication to a case of solutions that leading to the requirement of Big Rip or Little Rip singularity. Instead, from this case of solutions we should choose those requirement which the constraint by observational data. In other view point, some other types of singularity maybe appears in our work. one can easily check other type singularity with suitable conditions in this model.

Now we should describe what case of equation of state is studied for determine the character of mixed fluids. Typically in this model, we use three energy-momentum contents, 1: The ordinary matter 2: Dark energy describe by the scalar field 3: energy-momentum content which determine by a Lorentz violation vector field. Here we consider that ordinary matter has partial part of entire energy-momentum content. But for other contents, in our model it is possible to take the "trigger mechanism" to describe dynamical equation of state. i.e. we suppose that scalar- vector-tensor theory with LIV acts similar a hybrid inflation models. In this regard, scalar and vector field have the roles of inflaton and the "waterfall" field. Hence, we can fine-tuning parameters of model for best fit with observational data [10,25]. It is acceptable to expect that one of them suddenly dominate and we have a cosmological stage, for example, inflation phase or acceleration phase and etc. Note that an important result in this context is crossing of phantom divide barrier PDL that determine with a single minimal coupling scalar field and by a Lorentz violating vector field, if considering suitable fine tuning of model parameters.

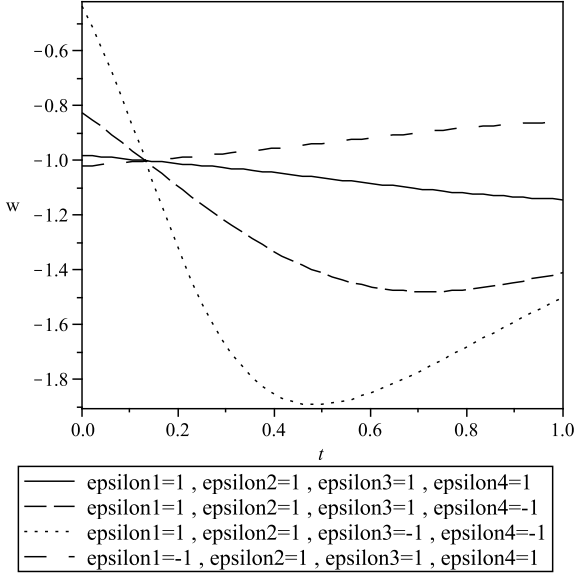


Figure 2: Crossing of phantom divide line.

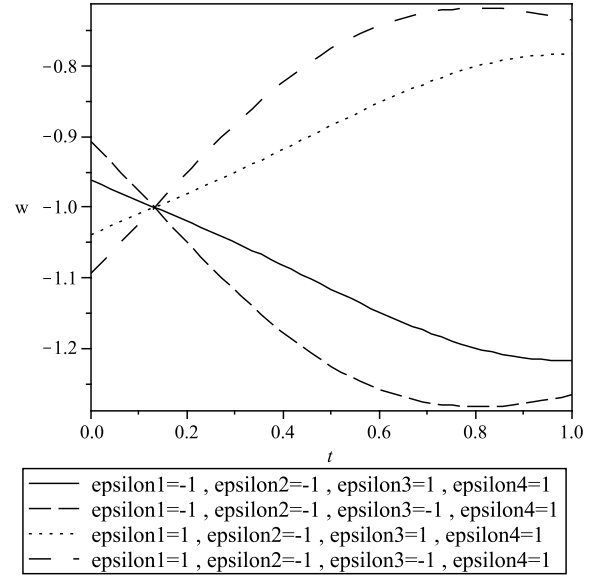


Figure 3: Crossing the phantom divide line.

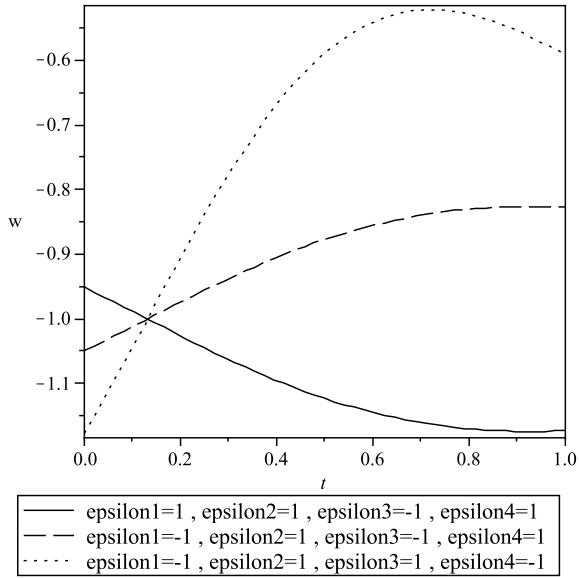


Figure 4: Crossing the phantom divide line.

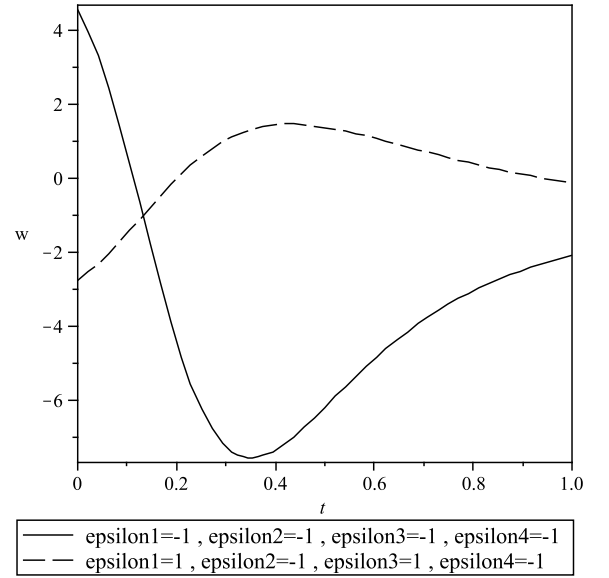


Figure 5: Crossing the phantom divide line.

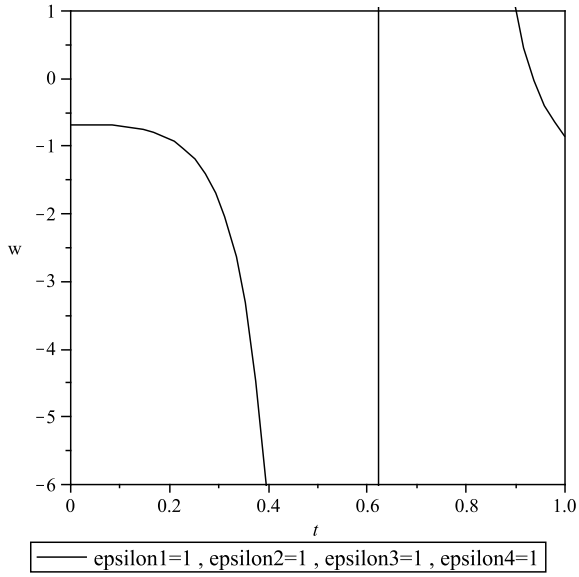


Figure 6: Rip singularity

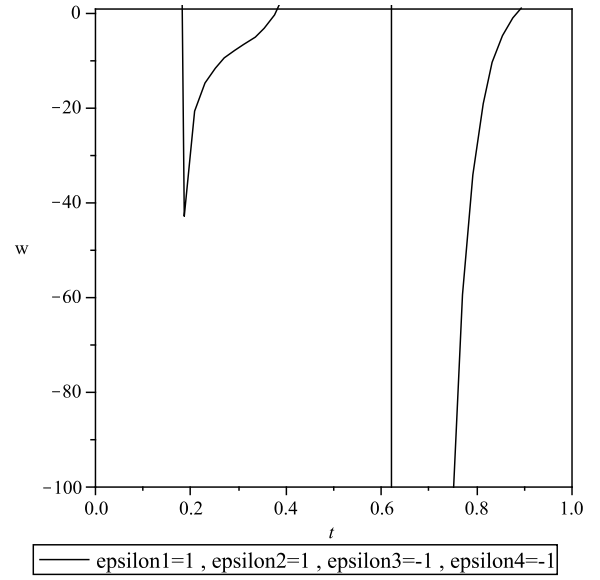


Figure 7: Rip singularity

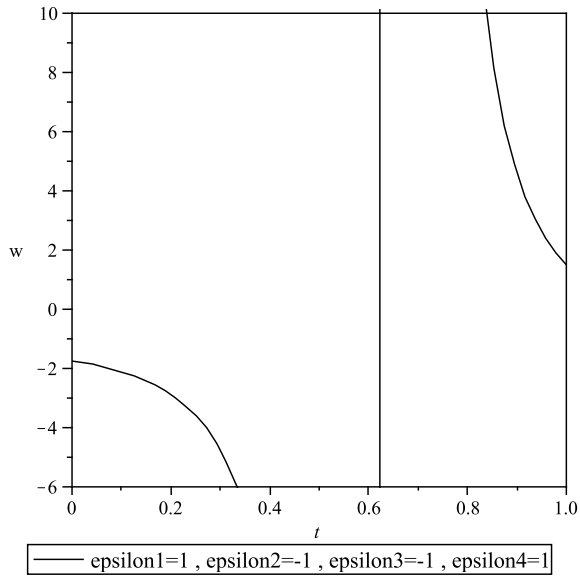


Figure 8: Rip singularity

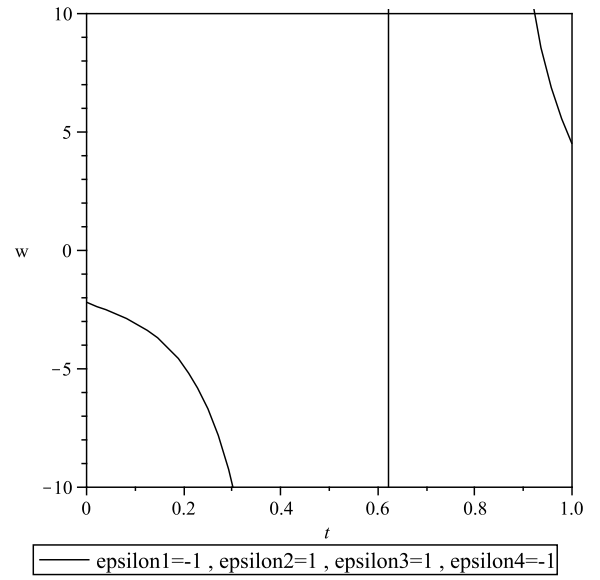


Figure 9: Rip singularity

Table 1: Considering Crossing phantom divided line and Rip singularity in equation of state in equation (36) on all branch for  $\epsilon = -1$  (middle) and  $\epsilon = +1$  (right) in equation (21) for potential of the form  $V(\phi) = \lambda'\phi^2$

$\epsilon_1 \ \epsilon_2 \ \epsilon_3 \ \epsilon_4$	Crossing PDL	Rip singularity
+ + + +	Yes	Big
+ + + -	Yes	Big
+ + - -	Yes	Little
+ - - -	Yes	Big
- - - -	Yes	normal
- + + +	Yes	Big
- - + +	Yes	Big
- - - +	Yes	normal
+ - + +	Yes	crossing PDL
+ - - +	Yes	Big
+ + - +	Yes	normal
+ - + -	Yes	Big
- + - +	Yes	Big
- + + -	Yes	Big

## 5 Summary

We have consider a possible violation of Lorentz invariance in a DGP Brane cosmology. Also we have shown that by a suitable choice of parameter space, it is possible to have crossing phantom divided barrier (PDL) and it can predicts Rip singularity in LIV context. We used an interactive picture, a minimally coupled scalar field and a Lorentz violating vector field can lead to the phantom phase. The comparison of our results with cosmology of 4D non-minimal vector theories show same aspects but with non minimal coupling in vector term, for example, explain the late time acceleration of the Universe[26,27,28]. Instead, those models have been shown that a non minimally vector field can be a candidate for dark energy[28] just like our model.

In other view point, in this framework, there is the possibility of a Rip singularity by suitable tuning in the parameters. As shown in Figs. 6-9, we have studied some type of solutions called Little or Big Rip, that maybe occur in structures of the Universe, according to the increasing of the dark energy density. However, In correct choice of parameters maybe other types of singularity appear in this model. Finally we emphasize that we have used a model of a scalar field with Lorentz violation to describe some cosmological aspects. We believe this model can perform some fascinating predictions for observations.

### Acknowledgement

We would like to thank an anonymous referee for his/her highly valuable comments.

## References

- [1] A. G. Riess, et al., Astron. J. **116** (1998) 1009 ; astron. J. **117** (1999) 707
- [2] S. Perlmutter et al, Nature **391** (1998) 51
- [3] M. Hicken et al., Astrophys. J. **700** (2009) 1097
- [4] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 330
- [5] W. J. Percival et al., Mon. Not. Roy. Astron. Soc. **401** (2010) 2148
- [6] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208
- [7] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **429** (1998) 263  
N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Rev. D **59** (1999) 086004

- [8] L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370  
L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690
- [9] S. Kanno and J. Soda, Phys. Rev. D **74** (2006) 063505
- [10] Arianto, Freddy P. Zen, Bobby E. Gunara, Triyanta and Supardi, JHEP **09** (2007) 048; Freddy P. Zen, Arianto, Bobby E. Gunara, Triyanta, A. Purwanto, Euro. Phys. J. C, **63** 3 (2009) 477-490
- [11] K. Nozari and S. D. Sadatian, JCAP **01** (2009) 005
- [12] S. D. Sadatian and K. Nozari, Europhysics Letters, **82** (2008) 49001
- [13] A. Vikman, Phys. Rev. D **71** (2005) 023515. See also other papers which accepted almost this viewpoint: Y. H. Wei and Y. Z. Zhang, Grav. Cosmol. **9** (2003) 307; V. Sahni and Y. Shtanov, JCAP **0311** (2003) 014; Y.H. Wei and Y. Tian, Class. Quantum Grav. **21** (2004) 5347; F. C. Carvalho and A. Saa, Phys. Rev. D **70** (2004) 087302; F. Piazza and S. Tsujikawa, JCAP **0407** (2004) 004; R-G. Cai, H. S. Zhang and A. Wang, Commun. Theor. Phys. **44**(2005) 948; I. Y. Arefeva, A. S. Koshelev and S. Y. Vernov, Phys. Rev. D **72** (2005) 064017; A. Anisimov, E. Babichev and A. Vikman, JCAP **0506** (2005)006; B. Wang, Y.G. Gong and E. Abdalla, Phys. Lett. B **624** (2005) 141; S. Nojiri and S. D. Odintsov, hep-th/0506212; S. Nojiri, S. D. Odintsov and S. Tsujikawa, hep-th/0501025; E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D **71** (2005) 103504; H. Mohseni Sadjadi, Phys. Rev. D **73** (2006) 063525; W. Zhao and Y. Zhang, Phys. Rev.D **73** (2006)123509; I. Ya. Arefeva and A. S. Koshelev, hep-th/0605085.
- [14] S. Nesseris, L. Perivolaropoulos, JCAP **0701** (2007) 018
- [15] V. A. Kostelecky and S. Samuel, Phys. Rev. D **40** (1989) 1886
- [16] P. H. Frampton, K. J. Ludwickl, and R. J. Scherrer, Phys. Rev. D **84** (2011) 063003
- [17] I. Brevik, E. Elizalde, S. Nojiri and S. D. Odintsov, appear in Phys. Rev. D; arXiv:1107.4642
- [18] P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov, R. J. Scherrer, Phys. Lett. B **708** (2012) 204-211
- [19] S. Nojiri, S. D. Odintsov, D. Saez-Gomez, arXiv:1108.0767v1

- [20] C.M. Will, Theory and Experiment in Gravitational Physics, (Cambridge Univ. Press, Cambridge, 1993); V. A. Kostelecky and S. Samuel, Phys. Rev. D **39** (1986) 683
- [21] S. M. Carroll and E. A. Lim, Phys. Rev. D **70** (2004) 123525
- [22] K. Nozari, JCAP, **09** (2007) 003
- [23] Kei-ichi Maeda, S. Mizuno and T. Torii, Phys. Rev. D **68** (2003) 024033
- [24] S. Tsujikawa, Phys. Rev. D **62** (2000) 043512
- [25] T. Mariz, J. R. Nascimento, A. Yu. Petrov, A. F. Santos and A. J. da Silva, [arXiv:0807.4999]
- [26] A. Golovnev, V. Mukhanov and V. Vanchurin, JCAP **0806** (2008) 009
- [27] T. S. Koivisto and D. F. Mota, JCAP **0808** (2008) 021
- [28] A. Golovnev, Phys. Rev. D **81** (2010) 023514
- [29] A. V. Astashenok<sup>1</sup>, S. Nojiri, S. D. Odintsov and A. V. Yurov, to appear in PLB, arXiv:1201.4056v2
- [30] A. A. Starobinsky, Grav. Cosmol. **6** (2000) 157; R. R. Caldwell, Phys. Lett. B **545** (2002) 23 ; R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91** (2003) 071301
- [31] E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D **70** (2004) 043539
- [32] S. Nojiri and S. D. Odintsov, Phys. Lett. B **686** (2010) 44
- [33] K. Bamba, S. Nojiri and S. D. Odintsov, JCAP **0810** (2008) 045
- [34] J. Barrow, Class. Quant. Grav. **21** (2004) L79
- [35] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71** (2005) 063004
- [36] P. H. Frampton, K. J. Ludwick and R. J. Scherrer, Phys. Rev. D **85** (2012) 083001.